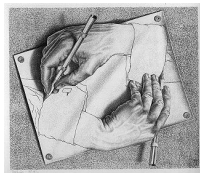# GL within HOL Light

Experiments on theorem provers within theorem provers

A project within the project "IT Matters: Methods and Tools for Trustworthy Smart Systems" (PRIN 2017FTXR7S)

Cosimo Perini Brogi
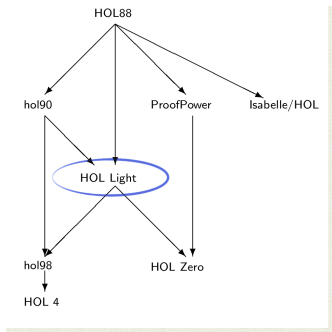IMT School for Advanced Studies Lucca

"Proof and Computation" Autumn School 2023

Picture credit: "The Magic of M.C. Escher", WikiMedia Commons

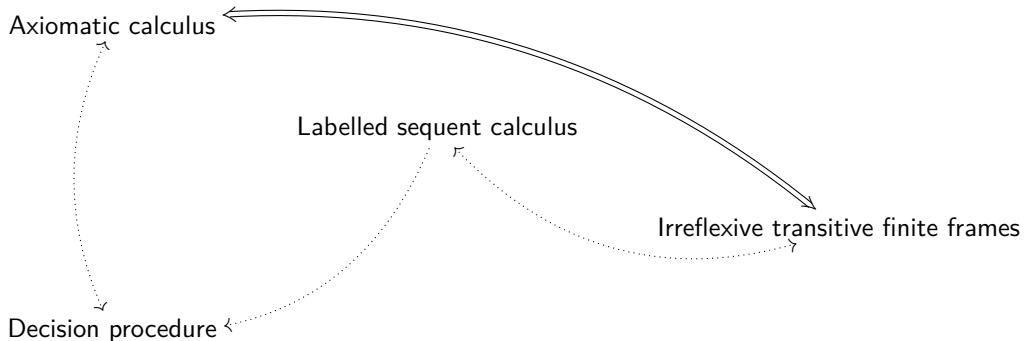# Brief glance at HOL Light
(Harrison 2017)

- ○ Clean logical foundations $\approx$ *Principia Mathematica*
- ○ LCF-style proof checker based on polymorphic simple type theory $\approx$ small class of *primitive inference rules* for creating theorems + *derived inference rules* to be programmed on top
  - $\Rightarrow$ 10 primitive rules
  - $\Rightarrow$ 2 conservative extension principles
  - $\Rightarrow$ Axioms of choice, extensionality, and infinity
- ○ Written as an OCaml program $\approx$ three datatypes for the logic: `hol_type`, `term`, and `thm`
- ○ Goal-directed proof development $\approx$ tactic(al)s + automated methods (in the appropriate domains)

Despite its simple foundations, HOL Light includes a large library of mathematical results in topology, analysis, Euclidean geometry, QBF, floating point algorithms, FOL, limitative results, . . .

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The current prototype

GL library

Axiomatic calculus

Labelled sequent calculus

Irreflexive transitive finite frames

Decision procedure

Code: https://github.com/jrh13/hol-light/, directory GL
Paper: *Mechanising Gödel-Löb provability logic in HOL Light*, J. Autom. Reasoning 67, 29 (Open Access)

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Short Demo

Many thanks for listening!